Puzzle of the Week

Fractions – 12

THE CHALLENGE: Use the numbers 1 to 9 at most once each in each set of boxes. First, make $\frac{\square}{\square} \times \frac{\square}{\square}$ equal to 2/3, and then find values that make $\frac{\square}{\square} \times \frac{\square}{\square}$ as close as possible to 5/11.

$$\frac{\square}{\square} \times \frac{\square}{\square} = \frac{2}{3}$$

$$\frac{\square}{\square} \times \frac{\square}{\square} \sim \frac{5}{11}$$

1 2 3 4 5 6 7 8 9

Puzzle of the Week

Fractions – 12 – Notes

THE CHALLENGE: I will write the expression as A/B x C/D to make it easier to talk about.

Part 1: A/B x C/D = 2/3. Multiply both sides by 3 x B x D, so this becomes 3 x A x C = 2 x B x D. Look at this using the prime factors 2 and 3 – these must balance on the two sides of the equation. From 3, 6, 9, we have four factors of 3 to work with. From 2, 4, 6, 8 we have seven factors of 2 to work with. There are two cases for handling the 3's.

- Case 1: Neither A nor C is 3 or 6, and B is 3 or 6.
    - B=3: 3 x A x C = 2 x 3 x D reduces to A x C = 2 x D. Solutions to this are 1 x 4 = 2 x 2, 1 x 8 = 2 x 4.
    - B=6: 3 x A x C = 2 x 6 x D reduces to A x C = 4 x D. The solution to this is 1 x 8 = 4 x 2.
- Case 2: A is 3 or 6, and B is 9.
    - A=3: 3 x 3 x C = 2 x 9 x D reduces to C = 2 x D - the options for (C, D) are (2, 1), (4, 2), and (8, 4).
    - A=6: 3 x 6 x C = 2 x 9 x D reduces to C = D, which is impossible.

So, the solutions to the problem are given by:

- (1 x 4) / (3 x 2)
- (1 x 8) / (3 x 4)
- (1 x 8) / (6 x 2)
- (3 x 2) / (9 x 1)
- (3 x 4) / (9 x 2)
- (3 x 8) / (9 x 4)

Part 2: A/B x C/D ~ 5/11. Multiply both sides by 11 x B x D to turn this into 11 x A x C ~ 5 x B x D. We want to find multiples of 11 that are close to multiples of 5 and that can be produced using 1 to 9.

Here is the list to consider. These are multiples that are 1 apart and are producible using 1 to 9: 11 x 4 ~ 8 x 5; 11 x 9 ~ 20 x 5; and 11 x 16 ~ 35 x 5. The bigger the numbers are, the better, so let's look at this last one.

(2 x 8) / (5 x 7) = 16 / 35 is very close to 5/11 - they differ by 1 / (35 x 11) = 1 / 385.

Perhaps the multiples that are two or three apart will work out better? There do not seem to be any candidates that would be an improvement.